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Supply Chain Coordination with Buy-back and Wholesale-price Contracts under Random Demand

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ABSTRACT
In order to increase supply chain total profit, we design a supply chain collaboration mechanism between one supplier and one retailer. Throughout the paper we present one supplier and one retailer set up with random demand in a decentralized supply chain. An incentive function on the buy-back cost and wholesale price cost is presented and scheme through the buy-back contract and wholesale-price contract has been developed to enrich the retailer take part in the collaboration ways. Furthermore, the paper shows that, how corporation between buy-back contract and wholesale contract could coordinate the supply chain in both a decentralized and centralized supply chain. Lastly, we set up numerical analyses and the result shows the implied collaboration mechanism is not permit the decentralized system to achieve the same performance as the centralized decision however it allows both members in the supply chain gains profit sharing by setting up the contract parameters.
Keywords – Supply chain, Collaboration mechanism, Buy-back contract, Wholesale contract.

I. INTRODUCTION

When we talk about supply chain collaboration with contracts, we start to review supply chain collaboration related research papers, wherein to design effective incentive schemes can allow the decentralized system of supply chain to achieve optimal performance, has been studied in several operations management literatures (Fawcett et al. 2012, Martin et al. 2010). Supply chain collaboration is discussed last decades by academics (Barratt et al. 2004, Fawcett et al. 2012, Lambert et al. 2004). In these papers related to pay attention to how to set up the effective coordination mechanism by different kinds of contracts. (Haoya et al. 2010) studied coordination contracts for supplier – retailer producing and selling fashionable products exhibiting a stochastic price independent demand (Giannocaro et al. 2004). A model of the supply chain contract with revenue – sharing mechanism and this model allows the system efficiency also it could improve the supply chain member’s profits by editing the contract parameters. Chaharsooghi and Heydari develop an incentive scheme according to credit option contracts to coordinate the reorder point and order quantity and improve the overall supply chain profitability as well as each member’s profitability (Chaharsooghi et al. 2010). Ozen et al. concentrate on coordination of manufacturing and the retailers with buy-back contracts and prove buy-back contracts; in general, the distribution system cannot achieve the same performance as the centralized system (Ozen et al. 2006). Apart from in accordance with what is done, this paper considers collaborations of a two-echelon supply chain consisting of one supplier and one retailer based on random demand and finds a collaboration mechanism with the buy-back contract and wholesale contracts which not only allows the decentralized system to perform just well as a centralized one, but also provides a option of cost allocation between the members of supply chain. This paper consists of following: In section II, the basic assumptions of the model are presented and the total profit functions of supplier and the retailer are discussed and also two extreme decentralized and centralized of supply chain system to evaluate the collaboration mechanisms above sections. Section III the collaboration mechanism with the buy-back and wholesale contracts under uncertain demand is developed. Numerical analysis is done in Section IV. We will conclude the paper in Section V.

II. Model and Decision Analysis

In our model we consider supply chain coordination with both members a supplier and a retailer in a one-period setting. The retailer discovers a stochastic customer demand x and he has to consider his stocking quantity that he orders from the supplier, at the beginning of selling-period. When placing his order q, exact demand is unknown for the retailer that realization but knows the distribution of demand. After a certain period of time, the goods are produced and shipped to by supplier to the retailer. Then the demand is realized and satisfied as much as possible.
Throughout the paper we use following notations:

$\prod^s$	Supplier's profit
$\prod^r$	Retailer's profit
S	Salvage cost for retailer
q	Order quantity for retailer
p	Selling price for retailer
c	Production cost for the supplier
w	Wholesale price
u	Shortage value for retailer

For the given value of the order quantity, supplier's profit is given by

$$\prod^s = (w-c)q \quad (1)$$

For the given value of the order quantity retailer's profit is given by

$$\prod^r = p\min\{q,x\} + S[q-x]^+ - u[x-q]^+ - wq \quad (2)$$

Where

$$[q-x]^+ = \max\{0, q-x\},$$
$$[x-q]^+ = \max\{0, x-q\}$$

When the retailer places his order, only the distribution of the demand $F(x)$ and the density function $f(x)$ are known. By Eq. (2), the expected profit for the retailer is given by

A. Supply chain decentralized decision

In the supply chain decentralized decision situation on the supplier and retailer optimize their objective functions individually and retailer defines optimal order quantity that achieves highest expected profit depending on the wholesale price of the supplier.

$$E(\prod^r) = p\int_0^q xf(x)dx + p\int_q^\infty qf(x)dx + $$
$$+S\int_0^q (q-x)f(x)dx - u\int_q^\infty (x-q)f(x)dx - wq \quad (3)$$

First derivative of function $E(\prod^r)$ on the order quantity is q solved by

$$\frac{dE(\prod^r)}{dq} = p\int_q^\infty f(x)dx + S\int_0^q f(x)dx + u\int_0^\infty f(x)dx - w \quad (4)$$

Let's the right of the Eq. (4) equals to zero then optimal order quantity in decentralized situation q^*_{dec} can be calculated solving the following equation:

$$F(q^*_{dec}) = \frac{u+p-w}{u+p-S} \quad (5)$$

That $F(x)$ represents a continuous probability distribution of the random variable x

B. Supply chain centralized decision

In the supply chain centralized decision situation the profit decision problem apart from decentralized decision can be defined as a joint optimal problem. Thus the actual total profit of the supply chain is given by

$$\prod^c_t = p\min\{q,x\} + \max\{h,S\}[q-x]^+ - u[x-q]^+ - cq \quad (6)$$

Wherein $\prod^c_t$ intends for the actual total profit of the supply chain and h intends for the salvage value for the supplier. The expected total profit function is given by

$$E(\prod^c_t) = p\int_0^q xf(x)dx + p\int_q^\infty qf(x)dx + $$
$$+\max(h,S)\int_0^q (q-x)f(x)dx - u\int_q^\infty (x-q)f(x)dx - cq \quad (7)$$

By solving the Eq. (7) the first derivative of function $E(\prod^c_t)$ on the order quantity q and let it equal to zero the supplier and retailer define the optimal order quantity q^*_{cen} able to get maximal expected profit for the entire supply chain.

$$F(q^*_{cen}) = \frac{u+p-c}{u+p-\max\{h,S\}} \quad (8)$$

Similarity between Eq. (5) and Eq. (8), normally $w \geq c$, $S \leq \max\{S,h\}$, thus it is without any doubt can be verified that the decentralized decision situation more order quantity will be placed by retailer then the decentralized decision situation, that leads to maximal total profit for entire supply chain. Hence it is necessary that collaboration mechanism would be set up to gain higher total profit of supply chain.

III. Supply chain collaboration mechanism

According to the analyses of supply chain centralized decision and decentralized decision represents that in the decentralized system scenario without collaboration the expected total profit of the entire supply chain is generally lower than in centralized scenario one. Salvaging at the supplier is more profitable than salvaging at the retailer because the supplier could redirect the unsold or not-needed units to the market of remanufacture and get positive revenue. This chance becomes significant if the supplier offers buy-back option to the retailer (Ozen et al. 2010). Hence here consider a variable buy-back cost that the supplier promises to offer to the retailer when the ordered amount of the retailer more than market demand at the end of the selling period. Let b denote the buy-back cost the supplier has to pay for every unit. Then the actual total profit of the supplier and retailer and also total profit of the entire supply chain after introducing buy-back cost are then given by

$$\prod_{col}^{s} = (w-c)q + (h-b)[q-x]^{+} \quad (9)$$

$$\prod_{col}^{r} = p\min\{q,x\} - u[x-q]^{+} - wq + b[q-x]^{+} \quad (10)$$

$$\prod_{t}^{ent} = p\min\{q,x\} + h[q-x]^{+} - u[x-q]^{+} - cq \quad (11)$$

By similarity between Eq. (6) and Eq. (11), its surely that total profit of supply chain in decentralized decision and the total profit of the entire supply chain after entering incentive function are different depending on the order quantity of the retailer salvaging at the supplier is more profitable than salvaging at the retailer, that is $h \geq S$

By Eq. (10) the expected profit of the retailer is given by

$$E(\prod_{col}^{r}) = p\int_{0}^{q} xf(x)dx + p\int_{q}^{\infty} qf(x)dx - u\int_{q}^{\infty}(x-q)f(x)dx$$

$$-wq + b\int_{0}^{q}(q-x)f(x)dx \quad (12)$$

After solving this first derivative of function $E(\prod_{col}^{r})$ on the order quantity q and let it equals to zero, the retailer defines the optimal order quantity that can get maximal expected profit. The optimal order quantity q_{col}^{*} can be calculated solving as following:

$$F(q_{col}^{*}) = \frac{u+p-w}{u+p-b} \quad (13)$$

Note that buy-back contract cannot make the distribution system can get the same performance as the centralized system. Hence, it needed to introduce wholesale price contract different from the buy-back contract realize the perfect supply chain collaboration. Let's accept that buy-back price b and wholesale price w that the decisions of the supplier purpose to get the higher total profit of the supply chain and the supplier directs to the retailer to choose overall optimal decision. To surely that the retailer chooses this value of order quantity that incurs overall maximal expected total profit, the supplier must fix the buy-back cost b and wholesale price w so that following equation holds:

$$F(q_{col}^{*}) = F(q_{cen}^{*})$$

$$\frac{u+p-w}{u+p-b} = \frac{u+p-c}{u+p-\max\{h,S\}} \quad (14)$$

After solving this Eq. (14) the optimal wholesale price w as a function of buy-back price b is achieved:

$$w^{opt} = u + p - \frac{u+p-c}{u+p-\max\{h,S\}}(u+p-b) \quad (15)$$

To ensure that in each combination the buy-back cost and wholesale price that satisfied the above equation that retailer chooses the overall optimal order policy that leads to maximal expected total profit of the entire supply chain. According to the supply chain members agree with collaboration model, after introducing incentive methods the expected profits of supplier and retailer might be more than that with decentralized decision situation, requires to fulfill as follows:

$$E(\prod_{col}^{r}) \geq E(\prod_{dec}^{r}) \text{ and } E(\prod_{col}^{s}) \geq E(\prod_{dec}^{s})$$

So far, it has been developed that collaboration mechanism that gain an overall optimal performance of the entire supply chain.

IV. Numerical analyses

In order to discuss the model and illustrate the conclusion more clearly, this section through numerical example analysis the supply chain decision results in both decentralized and centralized system are calculated and also get the better understanding collaboration model. In this section we use following parameters: wholesale price of the selling product is $w = 100\$$. The production cost for the supplier $c = 60\$$. For the retailer selling price per unit for the market $p = 130\$$ and shortage cost $u = 85\$$. For the retailer salvaging price per unit $S = 35\$$ and salvaging price for the supplier $h = 50\$$. Random variable x is on the market that we get the range of random variable from 100 to 200 to make easy numerical calculation. That density function is

$$f(x) = 1/(200-100) \quad 100 \le x \le 200 \quad (17)$$

Then probability distribution becomes:

$$F(x) = (x-100/200-100) \quad 100 \le x \le 200 \quad (18)$$

A. Decision Analysis of Supply Chain

From the Eq. (5) and Eq. (18) density function optimal order quantity of the retailer in supply chain decentralized decision can be solved as

$$\frac{q^*_{dec}-100}{200-100} = \frac{u+p-w}{u+p-S} \quad (19)$$

After this simple calculation the optimal order quantity q^*_{dec} becomes 163 units. Supplier expected profit is $E(\prod^s)$ is 6670 USD and expected profit of retailer $E(\prod^r)$ is equals to 2880 USD after that the total entire supply chain $E(\prod^{ent}_t)$ is equals to 9550 USD, Then, optimal order quantity of the retailer in supply chain q^*_{cen} is equal 193 units. Maximal expected profit of entire supply chain $E(\prod^c_t)$ is equals to 10542.5 USD. Table 1 is illustrated that expected supply chain total profit in both centralized and decentralized decision.

#	Decentralized decision	Centralized decision
q^*	163	193
$E(\prod^s)$	6670	-
$E(\prod^r)$	2880	-
$E(\prod^{ent}_t)$	9550	10542

Table 1. Supply chain decision analyses

B. Running Collaboration Mechanism Application

Optimal wholesale price as function of buy-back cost is given by

$$w^{opt} = 91 + 0.93b$$

Where the constraint on the buy-back cost is attained

$$E(\prod^r_{col}) \ge 2880 \quad E(\prod^s_{col}) \ge 6670$$

Buy-back cost is calculated from Esq.'s (9), (12) and (17)

$$96 \le b \le 104$$

The result of the total summarized numerical analyses is shown in Table 2. It's shown that if buy-back cost is increasing the wholesale price cost also increases as the expected profit of the supplier increase and retailer's expected profit is decreases.

b	w	$E(\prod^s)$	$E(\prod^r)$	$E(\prod^{ent}_t)$
97	104	6671	3872	**10542**
98	106	6808	3735	**10542**
98	107	6919	3623	**10542**
99	107	7048	3511	**10542**
100	108	7147	3396	**10542**
101	109	7246	3283	**10542**
102	110	7374	3169	**10542**
103	110	7502	3055	**10542**
104	111	7601	2942	**10542**

Table 2. Result of numerical calculation

V. Conclusions

This paper illustrated the problem of supply chain collaboration for the decentralized with one supplier and one retailer under uncertain and random demand. The buy-back and wholesale price contract model are used. In order to better understanding the collaboration model of supply chain the numerical analysis is done. This paper proved that by adjusting the parameters of buy-back and wholesale price contract design members lead the supplier and the retailer to increase profits comparing with decentralized supply chain by setting the parameters.

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